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from the Author

Photographic Notes and Formulæ:

BY

SIR DAVID SALOMONS,

M.A., F.R.A.S., F.C.S., Assoc. Inst. C.E., Mem. Inst. E.E.

Tunbridge Wells:

A. K. Baldwin, Grosvenor Printing Works.

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Preface.

THIS little book consists of a collection of Papers and Articles read and written by the Author at various times, together with other matter which is likely to interest the reader. In many instances the Papers and Articles have been much altered from the original publications, in order to extend or complete the matter treated upon.

Several subjects are touched upon which are of interest to the photographer, and which can be found in no other book. The Author required information of the kind here dealt with, and, being unable to find it elsewhere, he was obliged to work the matter out for himself; and now he offers the results to his fellow-workers, many of whom may have found themselves in a similar position.

BROOMHILL,

March, 1890.

CONTENTS.

HINTS FOR THE DARK ROOM.

Buy the dry plates in the winter for use during the following summer, and thus ensure satisfactory results. Keep all plates in tin boxes, which are sold nearly everywhere ; they are proof against damp, and light-tight under all conditions, which cannot be said of wooden and other boxes. They have also the advantage of taking up very little room, and can be opened easily in all weathers.

Let the dark room always be warm, never below 60° Fahr.. This may be done at very small expense with the gas stove called " Lux Calor," which requires no chimney, and gives off no bad fumes, such products, except the carbonic acid gas, being dissolved in the condensed water collected in these stoves.

Have no hyposulphite of soda in crystals, but keep a solution of 1 ounce hypo to 1 ounce water, which is practically a saturated solution. Dealers will sell it made this way. Some processes require a solution 1 part hypo to 4 of water, others 1 to 5, &c., so in a moment the desired solution is made without recourse to a number of bottles. The same may be said of oxalate of potash, the stock solution being 3 ounces water to 1 ounce oxalate of potash. The chloride of gold stock should be 1 grain to 1 ounce water. A sulphate of iron stock solution might also be made; others also, if desired. By this method a number of processes may be worked without confusion or taxing the memory, provided that on the wall cards are hung with the instructions applying to the various processes. These cards are made by mounting on cardboard the directions sent out by the makers, then they are sized and varnished, and rings attached (such as are used by shopkeepers to hang up tickets on their goods). By this means the instructions are always at hand, and not liable to become soiled by touching them with wet fingers.

Remember to have extra screens to the windows from May till September; these can be made of red tammy or paper stretched on frames and buttoned into the window frames, and thus be removable.

It is well to have all hypo dishes of a special material, say *papier maché* or celluloid; by this means no mistake can ever be made.

A Fletcher's radial burner is by far the best for heating

liquids, varnishing negatives, &c.; it is handy, and requires no stand for the pots or dishes. For platinotype these burners are invaluable. For boiling, making up solutions, &c., where hot water is required, there is no better way than to use a block tin saucepan with a loose porcelain inner pot, such as are sold for boiling milk. Large quantities of solution may be made up in this way without contact with metal, and the advantage of a block tin outer case is the freedom from rust. These vessels may be obtained to hold from 1 pint to 2 gallons.

When a plate is examined during development by lifting it from the tray to look through, lines are often caused by the developer running downwards. This evil may be avoided if the plate is continually turned when upright, and on returning the plate to the tray, flow the liquid in all directions.

The best way to varnish with spirit varnish is the following: —The room must be about 65° Fahr.; warm the plate as usual and flow the varnish. Do not rock the plate but place it, as held when draining, in a rack. After two or three minutes the crapeyness will disappear; it must then be heated again as usual. This method has two advantages: firstly, a more glassy surface; and, secondly, no chance of "firing" the plate, which often occurs if warmed directly the varnish has run off.

For solutions constantly required, such as hypo for fixing prints, acid solution for platinotype, &c., all the trouble of measuring large quantities may be saved by having a few large spare bottles, in which the quantities have once and for all been measured off and scratched with a diamond upon the bottle, so that it will in future only be necessary to fill with the various liquids up to the scratches.

In making starch for mounting prints, the supply of heat should be stopped the moment the liquid clears, otherwise much of the sticking quality will be lost. There is no better way of making starch paste than in the little tinned copper sold with the "Pendu light," or in a glue pot.

Albuminized prints should be rolled up face outwards to prevent the surface contracting.

Bisulphite of soda is made by pouring sulphuric acid upon sulphite of soda. Sulphurous acid fumes are given off during the process, and, this gas being dangerous to inhale, the mixing should be performed in the open air. For hydroquinone and eikonogen developments, 1 part liquid (or 1 part by weight, if crystals) bisulphite of soda to 4 parts hyposulphite of soda, and 20 parts water, makes the best fixing solution, the shadows being left clear and without stain.

STORING NEGATIVES,

AND

REGISTRATION OF NEGATIVES.

THE most convenient way to store negatives is to place them in packets of one dozen, each dozen being of the same size. The packets of 12 are then numbered. Woolf's, Tylar's, or any other maker's negative boxes are very useful for effecting this end.

An exposure book should always be used—it acts as an index. Each page in such a book should contain the particulars of 12 negatives, so that one page corresponds to one packet of negatives and becomes its index. Thus, a packet marked $\frac{4}{35}$ would indicate that the index to the 12 negatives is to be found in exposure book 4, at page 35, and conversely the number of the exposure book and page give the number of the packet in which those negatives will be found.

The method of entering negatives is here shown by a copy of a page such as an exposure book should have. The Author has published a book for this purpose. A sample page of the most convenient manner of entering lenses is also shown [see pp. 8 and 9].

Camera ....

No. of Slide	No. on Plate.	Date.		Time of day.	Light	Subject.	Lens.	Stop.	Seconds Expo-sure.	Shutter.		Devel-oper.	Result.	Name of Plate.	Remarks.
										Maker.	Setting.				
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Maker.										
Name of Lens.										
Equivalent Focal Length.	Stops.	Stops.	Stops.	Stops.	Stops.	Stops.	Stops.	Stops.	Stops.	Stops.
Remarks.										

ATMOSPHERE.

It is well known that distant objects, in clear weather, appear nearly of the same brilliancy, whether the distance is increased or diminished. But for this fact, the fixed stars would be invisible.

Putting the phenomenon in scientific language, it is to say that any object appears equally bright to the eye, whether near or far away. From observation this is not absolutely true, and air absorption is the reason why we do not see objects as bright at a distance as near. This air absorption produces what the artist calls "atmosphere."

If light passed through air with no loss in brilliancy, the objects which surround us would probably appear like so many electric lamps: in fact, we cannot conceive how matter would appear to the eye under such conditions. Damp, smoke, and dust in the air increase considerably the air absorption of light, and under aggravated conditions darkness is produced, as illustrated in a London fog. In the studio, air absorption (in ordinary states of the weather) is of little consequence, but for landscape work its presence is all-important.

But for this phenomenon, an outdoor scene would be impossible to render photographically, and indeed the brush could do it neither. The stereoscopic property of the eyes, which enables solids and distance to be so well appreciated, cannot be used when viewing an ordinary photograph or picture. This property is also of little or no service when viewing distant objects. In short, we judge distance to a great extent by the apparent brilliancy of the object looked at compared with others in the field of view. Experience has taught us the amount of brilliancy any object should possess if near at hand. Hence, in looking at a photograph we realize solidity partly by perspective and partly by the lighting, and distance almost entirely by the amount of brilliancy of the objects.

The practical meaning of "atmosphere" is very apparent from the above considerations, and the artistic meaning is that the lighting of the various objects in the picture are fairly proportional to their distance from the individual looking at the picture. Artists and others use the expression "atmosphere," it is feared, without knowing the true definition of the word. The word may be defined as "an air absorption of light proportional to distance, and sometimes an air absorption of light not proportional to distance, due to layers of air containing particles which absorb more light than other layers looked through." In the latter case it is almost impossible to render all distances to appear proportionally correct, unless there are breaks in the different absorbing layers of air, so as to obtain true renderings in some part of the picture.

"LARGE HEADS" IN THE STUDIO

To those who are accustomed to take small portraits, the difficulties which attend the production of large ones cannot be appreciated. They are very numerous, and some of them are almost impossible to overcome. When passing through the show-room of a first-class professional photographer, heads approaching life-size are always to be seen, and they appear very perfect. Those behind the scenes know well how necessary it is to pick and choose before such specimens can be found, and what a large amount of retouching was necessary both upon the negative and print before a good picture resulted. The special points which claim attention and give trouble are the following:—

1. A good subject.
2. The lighting and posing.
3. The lens.
4. The focussing.
5. The plate.
6. The developing.
7. The paper to print on.

Let us deal with these points in the above order.

1. It is evident that when the head approaches life-size, every imperfection in the subject makes itself evident. In two-inch heads the trouble commences, but it is small compared with that of larger sizes. Every irregularity of the skin throws a shadow of its own, which is not seen except by close inspection, and unfortunately the lens is most searching, reproducing every defect in a more marked manner than is apparent to the eye; and unseen marks frequently are discovered by the lens. Consequently, sitters having good features and a good texture of skin, with no markings, give the best pictures; it is desirable, therefore, only to attempt on a large scale the portraits of such persons. Artists generally make "life-size heads" four-fifths to nine-tenths of actual size, and, since these dimensions give the most pleasing results, photographers might well follow the rule. Again, it is useless to attempt the portrait of anyone who cannot "sit" a long exposure, for this is essential for good large work. Large plates may rank from 18 × 16 upwards.

2. The lighting of a four to eight-inch head must be very

subdued, or too great a contrast will be produced; but the whole face must be light and shade, with no large patches of either. To accomplish this, the top light must be well in advance of the sitter, and reflectors of white card placed at various points till the desired effect is produced. It is well to look at the sitter from the lens, placing the head in front of it, and thus see the object as the lens perceives it. One of the most troublesome things to do is to place the sitter's eyes right, and this may easily be done by the use of a stand having a coloured ball on the top which can be raised and lowered. This apparatus may be moved about, whilst the sitter looks at the ball, till the eyes appear satisfactory, and during the exposure he or she must look at the ball.

The pose requires great attention, and many might think this easy because the head only has to be placed. However, a very little experience will prove the error, for the height of the camera and many other points can destroy the proper perspective. For small heads the lens should look down slightly; for large ones it is best placed on a level, and the head slightly down in order to get the nose in good proportions.

3. The lens must have a long equivalent focus, say over 30 inches, or very little "depth" will be obtained; a small stop will also be found necessary. Even then it will be no easy matter to get the whole head in good focus. The camera has to be extended to nearly twice the equivalent focal length of the lens, so on this account alone the exposure will be nearly four times that given with the same lens for full length on the same size of plate. Then the lens, having a long focal length compared with the open aperture, is slow to work with at best, and the small stop does not diminish these difficulties. Hence, to expose fully, the "large head" requires nearly six times that which would be required for a full length picture, and no satisfactory result can otherwise be obtained. So a good light is very necessary.

4. There are many dodges of breaking through some of these troubles, so as to employ a shorter focus lens, a larger stop, or both. They are all better avoided if possible. One is to separate the lenses in the back combination of a portrait lens and produce thereby a slight positive aberration, commonly called "diffusion of focus." Another is to effect this during exposure; a third is to alter the extension of the camera slightly during exposure and thus get successive planes in true focus. The diffusion methods rarely produce such good results as can be obtained by its absence, and the extension plan is very limited, for the size of the image is slightly altered for each alteration in the length of the camera. No doubt in able hands both methods succeed if not abused.

It is a great mistake to get the whole head in good focus. A slight want of sharpness in those parts most removed from the camera is agreeable if properly managed.

5. Large plates appear to be more difficult to obtain without flaws and other faults than small ones. It is evident that the most rapid kinds should be employed, and the glass should be stout to enable it to be handled without fear of breaking, and not to bend under the pressure of the springs in the dark slide. Strong glass is essential for printing from, for if weak and not flat it is sure to crack in the printing frame. The edges ought to be ground, otherwise unpleasant cuts may result, the weight in the hand being considerable.

6. The developer should be very weak at starting, about one-fourth usual strength, and the pyro, if this developer is used, should be flowed without the ammonia to give a chance of removing all air bubbles which may form. Soaking in water first should be avoided. The ammonia is cautiously added, and by degrees the solution may be made stronger as the development advances.

Such negatives must not be made as dense as smaller ones for two reasons. Firstly, the lighting of the subject has been much softer than for small pictures, and secondly, albuminized paper will probably not be chosen to print on.

Still the negative should not be weak, but a happy medium selected. A large amount of retouching is indispensable for such subjects, and the coarser it is done the more beautiful the print, since it will have vigour instead of flatness, even though the pencillings may show. To make up for the subdued light in which the model was placed, a few sharp properly-placed high lights may be put on the negative with great advantage, for it is the picture we want, without inquiring into how the charms were produced. If the head is a mere study, likeness need not be preserved; in other cases great care is required not to lose it in retouching.

7. All the "mat" processes give more pleasing results than ordinary albuminized paper. Platinotype is excellent for such productions. The special paper prepared by the well-known firm of Messrs. Marion, under the name of coaguline, gives charming pictures. This paper is a mat albuminized paper, is toned and fixed, as is usual with such paper. If good drawing paper prepared as coaguline is employed, and the head is vignetted (by means of a mask of *papier minérale*) close to the outline, the result is more like an engraving than a photograph. If such a picture has a few touches of crayon and white put upon it, a drawing is apparently produced with half an hour's work.

There is no doubt six-inch heads and upwards are coming into favour, but, to secure a lasting desire on the part of artists and the public for such photographs, retouching upon the negative as well as the print must no longer be fought against, since it is absolutely essential for the production of good and pleasing pictures. With small negatives, or large ones full of small objects, retouching may be reduced to a minimum, but sensible people can appreciate that what is abuse in one case may be a virtue in another.

EXPOSURE.

No good print can be made from a bad negative. Amateurs constantly complain of their plates, find the paper will not tone, and meet with other difficulties, most of which, upon investigation, would be found to originate with faulty negatives.

To make a good negative, three things are necessary :—

1. Good lighting of the subject.
2. A correct exposure.
3. Development carried to the proper degree.

Let us consider No. 1 first. Many photographers fail to understand why some particular artist's productions are always so good, and so superior to their own, especially when they find their own negatives perfect by the usual standards. The reason for this superiority is generally to be found in the lighting of the object. This question seems simple to all except those who have made a study of the subject. In fact, the treatment of the object is one which requires much attention and study. A few persons have the "sense" of proper lighting born in them, but the majority must work for the knowledge, whilst some seem unable to appreciate differences produced by variations of light and shade, however much they try. In all cases the same object, well or badly lighted—all other things being equal—produces negatives so different that, judging by results, no one would believe that the lighting was the only difference made.

No. 2.—This is the chief difficulty with amateurs, and a plan is suggested which, although requiring modifications in practice, is not bad for general work. It is supposed that the stops are marked on the decimal system, for this method is very easy for calculation. If required to convert the decimal to the uniform (or Photographic Society's) standard, multiply the number by ten and divide by sixteen. The decimal system has the advantage that whole numbers are always employed in the method of calculating the exposure to be proposed.

In order to mark the stops correctly, the focal length of the lens must be known. The following is a very simple way of determining the focal length sufficiently near for all practical purposes:—Purchase a number of single lenses, commencing with one having three inches for its focal length. Those following should have focal lengths advancing by a quarter of an inch up to six inches, then by half inches to ten, and after this by inches to twenty. This gives a series of thirty-one lenses of, say, one inch diameter, which may be purchased at from one penny to twopence each at any optician.

This five shillings' worth of glasses gives the means of determining the focal lengths of all lenses, covering from quarter-plate to 12 × 10. To test a photographic lens consisting of more than one combination, hold one of the single lenses level with or touching the diaphragm slot, and throw an image of some object with the lens under observation on a sheet of paper. At night a candle or lamp flame answers well. By changing the single lenses till two images are shown, one by the photographic lens and one by the single lens, both in focus and equal in size, the required result is obtained, and whatever is the focal length scratched on the test lens it is also the approximate focal length of the photographic lens. When single combinations are to be tested, the single glasses must be held in a plane with the lens in the mount. By this method, in less than a minute, three or four lenses may have their focal lengths determined close enough to mark the stops.

The focal length of a lens being determined, measure the diameters of the stops in sixteenths of an inch. Now divide the focal length by the diameter of each stop, square the result, and put a decimal point (equivalent to dividing by ten) to each result. The number thus found for each stop should be stamped upon it, and may in future be regarded as the multiplier.

To find a starting-point, let us imagine that we expose with a standard lens—that is, one which would have "1" stamped on the stop. We may accept that one to five seconds exposure is sufficient with a standard lens indoors for almost every subject which is usually taken with general lighting.

For outdoors take one-tenth to five-tenths of a second. If the light indoors is good and the subject not exceptionally dark, for the months of May, June, July, August, and September, one second is sufficient; as the winter comes on, the time should be doubled or trebled, and in December quadrupled. Hence, all that is necessary is to determine whether one or two seconds should be given (for indoors this is the most common case), and then multiply by the number on the stop employed. Very rarely will a failure result if this plan is followed. Many other circumstances require to be taken into account on special occasions, but these will be entered upon in the next article.

Experienced photographers, and those who always take very similar subjects, can judge the exposure by the appearance of the picture upon the screen; but the method is crude and often leads to false results, only to be corrected by careful development.

The imaginary one to five seconds' exposure holds good only with very rapid plates; with slow ones the time must be doubled or more, which can be determined by a trial.

No. 3.—Having settled the questions of lighting and exposure, development becomes easy enough. In the first place, the developer may remain on a long time, for there is little fear of over-development, and none on the other side. In fact, as soon as the lights appear perfectly opaque by transmitted light, and the image shows at the back of the plate, the work is done. The image comes through very soon with cheap plates, so in these, density is the chief point to attain. Hence, if No. 1 and No. 2 have been properly carried out, No. 3 becomes a purely mechanical process, devoid of all those delicacies insisted on in text-books, and which are necessary when lighting and exposure are not correctly arranged.

"The proof of the pudding is in the eating," and the writer only settled this method of exposure after laborious experiment, and then put it in practice upon himself and on photographic beginners, which ended in all obtaining properly exposed negatives in every case.

RULES FOR EXPOSURE.

Exposure is regulated by the time of year, hour of the day, light, subject, lens employed, and speed of plate.

The difficulties of exposure may be completely overcome by employing the Decimal standard in place of the Photographic Society's standard, which is illogical and inconvenient. The former starts with a standard lens $\frac{f}{\sqrt{10}}$, whose relative exposure is 10 and called Standard 1. The second starts with a lens $\frac{f}{4}$, whose relative exposure is 16 and called Standard 1. It is evident that division by 10 is far simpler than by 16, for in all cases of comparison the division must be made, and dividing by 10 is simply to place a decimal point. The chief troubles of exposure are removed by marking the stops of all lenses to the Decimal standard, for there is then no need to tax the memory or make any further calculation.

Now, the standard lens by the Decimal system is practically Dallmeyer's 2B Patent, which is in the hands of most photographers, and its imitation is probably possessed by those who cannot afford the original. It is therefore fair to assume that no photographer, worthy of the name, exists who does not know the exposure with this lens open aperture, with a good light indoors and a quick plate. The full exposure in a well-lighted studio in summer under these conditions is about one second. We have therefore a foundation to start with, and this *one second* must be multiplied by the number of the stop used (no matter what lens is employed, if marked by the Decimal standard); also account must be taken of time of year, hour of the day, subject and plate. For out-of-doors the 1 second should be replaced by $\frac{1}{10}$ second. The following are very simple tables and complete enough for some of these factors, the assumption being fairly bright weather. If dull, allowances must be made for which no rules can exist, since the circumstances vary so much; but bright clouds and clear sky are about equal in light.

The relative value of the light for the time of year is approximately—

April, May, June, July, August	light	...	1
February, March, September	,,	...	2
January, October, November	,,	...	3
December	,,	...	4

At all times of the year the light is fairly equal from 11 a.m. till 2 p.m.

For relative value of subjects :—

Sea and sky	1
Landscape	4
Dark buildings and foliage	10
Indoor subjects well lighted ...	10 to 50
Dark subjects, interiors, and dark shadows	100 ,, 400

If the most rapid plates in use are termed 1, then a trial will determine relative value of any other plate. We have now all the data for an exposure: thus for out-door work— $\frac{1}{10}$ second $\times$ stop number $\times$ time of year $\times$ light at the time $\times$ subject number $\times$ plate number.

For example :—

Let stop number be 15.

,, time of year be May : number is 1.

,, light be dull, say $\frac{1}{2}$ if bright : this is therefore 2.

,, subject be exterior of an old farmhouse : number is 10.

,, plate used be ordinary or half most rapid : number is 2.

Then we get for exposure—

$\frac{1}{10}$ second $\times$ 15 $\times$ 1 $\times$ 2 $\times$ 10 $\times$ 2 = 60 seconds, or 1 minute.

Since there is a great margin allowable in exposures, the above answer may be much increased or diminished without injuring the result. It must be borne in mind that the stop number is only correct for solar focus, and approximately correct for landscape and small work; but when the camera is extended much beyond the solar focus of the lens, as is often done when copying and taking large heads, &c., the correct stop number is found, to a fairly accurate degree, thus :—

Divide distance in inches between stop slit and ground glass by diameter of stop in inches, square result and divide by 10 : this gives the number on the Decimal system.

A Stanley's actinometer is very useful to ascertain the actinic value of the light; but it must be remembered that such instruments are only of service if referred to habitually, so as to have comparative values always before the mind.

RELATIVE RAPIDITY, RATIO,

AND STANDARDS.

1.—An aperture will permit light to pass through it in proportion to its area; and the area of a circle varies as the square of the diameter. Consequently the light which passes apertures of various diameters will be proportional to the squares of their diameters. If A and A$'$ = two aperture diameters, then the light passing through them will be as A^2 is to A'^2.

2.—Since the light falling upon a screen from a source of illumination varies as the square of the distance of the source of light from the screen, so therefore will the quantity of light upon the focussing screen vary as the square of the distance between the lens and the screen. In this case the lens acts as the source of light. Hence the light upon the ground-glass screen of a camera varies as the square of the focal length of the lens, *i.e.*, as f^2.

3.—Since the quantity of light which passes through the lens varies as A^2, and because the larger A the more light passes, and because the quantity of light falling upon the screen varies as f^2, the greater is f the less light upon the screen, it follows that for any aperture and any focal length the light falling upon the screen varies as $\frac{A^2}{f^2} = \left(\frac{A}{f}\right)^2$. If the aperture A be reduced to unity by dividing the numerator and denominator of the fraction $\frac{A}{f}$ by A, and let $\frac{f}{A} = m$, then we obtain $\frac{1}{m^2}$ for expressing the amount of light which falls upon the screen, not as a definite quantity, but a comparative one; that is to say, if several apertures and several lenses are to be compared, the light falling upon the screen in each case will be $\frac{1}{m^2}$ to $\frac{1}{m_1{}^2}$ to $\frac{1}{m_2{}^2}$, &c., where the several values m, m_1, m_2, have been found for each lens in the manner indicated.

4.—The only convenient way of expressing the functions of a lens is to consider the relation between its aperture and focal length, *i.e.*, $\frac{A}{f}$.

The focal length is a variable quantity, so it is usual to consider f in the above fraction to have a value equal to the focal length for parallel rays, such as solar rays; hence this focal length is called "solar focus," but strictly the "principal focus."

The fraction $\frac{A}{f}$ can always be reduced to the form $\frac{1}{m}$ and is usually written f/m, or $f\text{-}m$, or $\frac{f}{m}$; but it must be borne in mind that f/m is only a conventional way of writing $\frac{1}{m}$ when a lens is referred to.

5.—This expression f/m is termed the ratio (being in fact the relation between the aperture and focal length), written R for short: thus, $R = f/m$.

Since the amount of light falling upon the screen varies as $\frac{1}{m^2}$, or as m^2, which is the same thing; m^2 expresses the relative rapidity of one lens with any other lens, or the same lens with varied aperture.

Consequently we have a convenient means of expressing the relation existing between the aperture of a lens and its focal length, namely R (the ratio), which is in practice a fraction with photographic lenses (f/m); also we have a method of expressing relative rapidity.

Now find a way of measuring absolute rapidity, which means comparison with some standard lens, the choice of which is arbitrary.

6.—The Photographic Society of Great Britain has adopted a lens $f/4$ as the standard with which to compare all lenses. Such a lens has a relative rapidity of 16, *i.e.*, 4 squared. Hence it will be necessary to divide the comparative rapidity of any lens under examination by 16, to obtain its rapidity compared with the standard lens. For example, How much more rapid or slower is a lens $f/7$ than $f/8$?

Comparative rapidity of lens $f/7$ is 49;
" " " $f/8$ is 64;

Now divide each by 16—

$$\frac{49}{16} = 3{\cdot}06 \qquad \text{and} \qquad \frac{64}{16} = 4.$$

Hence the rapidity of lens $f/7$ is to that of lens $f/8$ as 3·06 to 4, and the former lens is consequently about a third more rapid than the latter one. The results 3·06 and 4 are the

standard numbers; they are also comparative rapidities, since the operation of squaring was gone through in obtaining these figures.

It is usual, however, to consider the number before dividing by 16 as the comparative rapidity number, and after division as the standard number. Consequently a lens $f/7$ has a comparative rapidity of 49, and a standard number 3·06.

If the behaviour of a lens $f/4$ is known, the conduct of any other lens is also known when the standard number is found.

In practice, the following is the method to find the standard number:—"Square the denominator of ratio, when expressed as a fraction with unity for numerator, and divide by 16; the result is the standard number on the system of the Photographic Society of Great Britain, generally called 'the uniform standard.'"

Dividing by 16 is an awkward process, and leads to results (in many cases) which have several decimal places. This led the late Mr. Dallmeyer to find a simple system, termed "the decimal system"; and all the lenses made by Messrs. Dallmeyer are marked upon this system.

7.—The decimal system starts with a lens $\frac{f}{\sqrt{10}}$ for the standard, and its relative rapidity is therefore 10. When comparing lenses it is only necessary to put a decimal point after squaring m, which indicates a division by 10. For instance, take the old case of two lenses, $f/7$ and $f/8$; the squares of 7 and 8 are 49 and 64. Now divide by 10, and we have 4·9 and 6·4 for their standard numbers. Nothing can be simpler. Dallmeyer's 3B with open aperture is practically a standard lens, being $f/3$.

8.—The question arises how to convert one standard to the other.

Let S represent the decimal system number.
,, P ,, uniform ,, ,,
,, R ,, ratio, which for photographic lenses is always a fraction, since the aperture is always less than the focal length.
,, f ,, equivalent focal length.
,, m ,, a number.

$$\text{Then} \quad P = \frac{1}{16\,R^2}$$

$$\therefore \quad R = \frac{1}{4\sqrt{P}} \qquad \text{(I.)}$$

$$\text{also,} \quad S = \frac{1}{10\,R^2}$$

$$\therefore \quad R = \frac{1}{\sqrt{10\,S}} \qquad \text{(II.)}$$

$$\text{hence,} \quad \frac{1}{4\sqrt{P}} = \frac{1}{\sqrt{10\,S}}$$

$$\therefore \quad 16\,P = 10\,S$$

$$\text{or} \quad 8\,P = 5\,S \qquad \text{(III.)}$$

$$\therefore \quad P = \cdot 625\,S \qquad \text{(IV.)}$$

$$\text{and} \quad S = 1\cdot 6\,P \qquad \text{(V.)}$$

From these equations all solutions can be found.

THE DISTANCES

BEYOND WHICH ALL WILL BE IN FOCUS.

The Author found a simple formula for ascertaining the distance beyond which all objects will be in focus with any given lens—it is $f + 100\ f^2$ R inches—where f and R have the same meanings as usual. This formula is very useful to ascertain the most suitable lenses for detective cameras, and for other purposes. It is assumed that, if points in the object are represented in the image by circles having diameters $\frac{1}{100}$ inch and less, the picture will be sharp.

A special instance will be given to show its use: $100\ f^2$ R inches may be written $8\frac{1}{3}\ f^2$ R feet, since 100 inches $= 8\frac{1}{3}$ feet. The addition of f in the formula may be neglected, being small compared with $100 f^2$ R. Most of the rapid landscape lenses work at $\frac{f}{8}$, so the formula reduces itself to f^2 feet approximately. This shows that, with any lens $\frac{f}{8}$, all will be in focus after a distance expressed by the square of its focal length in feet. Putting the formula in words, $100\ f^2$ R inches reads: 100 multiplied by the ratio, multiplied by the square of the equivalent focal length of the lens, expresses in inches the distance beyond which all objects will appear in focus. To give an example:—After what distance will all objects be in focus with a lens stopped $\frac{f}{12}$ (*i.e.*, $\frac{1}{12}$), the equivalent focus being 10 inches?

Answer—

$100 \times \frac{1}{12} \times 10 \times 10 = \frac{10000}{12} = 833\frac{1}{3}$ inches $= 69$ feet $5\frac{1}{3}$ inches. To be strictly accurate, add f ($= 10$) to this, and distance becomes 70 feet $3\frac{1}{3}$ inches from optical centre of lens.

It is a good plan to mark the stops for the distance after which all will be in focus, as well as the standard number. In practice this is valuable, for, when a subject is to be taken requiring a particular stop to get the focus right, it is found that the aperture is too small for the lighting, the attempt will not be made, and a plate is thus saved.

The following table will be found useful for many purposes. Some 1200 calculations were made to compile the results given.

FOCUS TABLE FOR DETECTIVE CAMERAS

AND OTHER SPECIAL WORK.

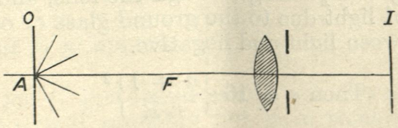
Focus of lens in inches.	Decimal Standard Stop Numbers.								Ratio marked on Stops.														Focus of lens in inches.
	5	10	15	20	25	30	35	40	$\frac{f}{7}$	$\frac{f}{8}$	$\frac{f}{9}$	$\frac{f}{10}$	$\frac{f}{11}$	$\frac{f}{12}$	$\frac{f}{13}$	$\frac{f}{14}$	$\frac{f}{15}$	$\frac{f}{16}$	$\frac{f}{17}$	$\frac{f}{18}$	$\frac{f}{19}$	$\frac{f}{20}$	
	No. of feet and inches after which all in focus.								Number of feet and inches after which all in focus.														
	' "	' "	' "	' "	' "	' "	' "	' "	' "	' "	' "	' "	' "	' "	' "	' "	' "	' "	' "	' "	' "	' "	
4	19·4	13·8	11·5	9·10	8·9	8·0	7·5	7·0	19·4	17·0	15·1	13·8	12·5	11·5	10·7	9·10	9·2	8·8	8·2	7·8	7·4	7·0	4
4¼	21·10	15·4	12·10	11·1	9·10	9·0	8·4	7·10	21·10	19·2	17·0	15·4	14·0	12·10	11·10	11·1	10·4	9·9	9·2	8·8	8·3	7·10	4¼
4½	24·5	17·3	14·5	12·5	11·1	10·1	9·4	8·9	24·5	21·5	19·1	17·3	15·8	14·5	13·3	12·5	11·7	10·11	10·3	9·9	9·3	8·9	4½
4¾	27·3	19·2	16 0	13·9	12·4	11·3	10·5	9·9	27·3	23·10	21·3	19·2	17·5	16·0	14·10	13·9	12·11	12·1	11·5	10·10	10·3	9·9	4¾
5	30·2	21·3	17·9	15·3	13·7	12·5	11·6	10·10	30·2	26·5	23·6	21·3	19·4	17·9	16·5	15·3	14·3	13·5	12·8	11·11	11·4	10·10	5
5¼	33·3	23·4	19·6	16·10	15·0	13·9	12·8	11·11	33·3	29·1	25·11	23·4	21·3	19·6	18·1	16·10	15·9	14·9	13·11	13·2	12·6	11·11	5¼
5½	36.5	25·8	21·5	18·5	16·5	15·0	13·10	13·0	36·5	31·11	28·5	25·8	23·4	21·5	19·10	18·5	17·3	16·2	15·3	14·5	13·8	13·0	5½
5¾	39·10	28·0	23·5	20·1	17·11	16·5	15·2	14·3	39·10	34·11	31·1	28·0	25·6	23·5	21·8	20·1	18·10	17·8	16·8	15·9	14·11	14·3	5¾
6	43·4	30·6	25·6	21·11	19·6	17·8	16·6	15·6	43·4	38·0	33·10	30·6	27·9	25·6	23·6	21·11	20·6	19·3	18·1	17·2	16·3	15·6	6
6¼	47·0	33·0	27·7	23·9	21·2	19·4	17·10	16·9	47·0	41·2	36·8	33·0	30·1	27·7	25·6	23·9	22·2	20·10	19·8	18·7	17·7	16·9	6¼
6½	50·10	35·9	29·10	25·8	22·10	20·11	19·3	18·1	50·10	44·6	39·7	35·9	32·6	29·10	27·7	25·8	24·0	22·6	21·3	20·1	19·0	18·1	6½
6¾	54·9	38·6	32·2	27·8	24·8	22·6	20·9	19·6	54·9	48·0	42·9	38·6	35·0	32·2	29·9	27·8	25·10	24·3	22·10	21·7	20·6	19·6	6¾
7	58·11	41·5	34·7	29·9	26·6	24·3	22·4	21·0	58·11	51·7	45·11	41·5	37·8	34·7	31·11	29·9	27·9	26·1	24·7	23·3	22·0	21·0	7

UNIVERSAL FORMULA
FOR
CALCULATING EXPOSURES FOR ENLARGING AND REDUCING.

Let I = the area of image (*i.e.*, the sensitised paper or plate).
„ O = the area of object (*i.e.*, the negative).
„ F = focal length, or conjugate, measured from O to the optical centre of lens, in inches.
„ f = solar focus of the lens in inches.
„ K = a constant multiplier, to make up for loss of light in passing through the medium (if any) placed between the light and negative, and in passing through the lens.
„ C = contact exposure in seconds, supposing that the medium (if any) placed between the light and negative is removed, and the sensitised paper or plate is placed in contact with the negative, all other conditions remaining the same. (It is evident that, by regulating the quantity of light, and varying its distance from the negative, C may be made any value, and can be easily determined by experiment.)
„ e = required exposure in seconds.
„ E = required exposure in minutes.
„ D = diameter of stop employed in inches.
„ n = number of diameters of I contained in diameter of O.
„ S = decimal standard value.
„ R = ratio, or $\frac{D}{f}$.

Now consider the following diagram, which speaks for itself.

Let light fall on O, and confine our attention to a point of light at A; this ray, on passing through O, spreads out in all directions, and only a cone of these rays traverses the lens, limited by the size of the stop shown:—



The relation between the whole of the rays and this cone may be thus expressed. Suppose a hemisphere to exist on

the lens side of O, with A as centre and F as radius; then it is evident that

$$\frac{\text{Whole of light}}{\text{Light passing lens}} = \frac{\text{Surface of hemisphere}}{\text{Portion of surface of hemisphere bounded by stop aperture.}}$$

$$\text{The numerator} = 2\,\pi\,F^2$$

$$\text{and the denominator} = \pi\,(\tfrac{1}{2}D)^2$$

(for the area of the stop aperture and the portion of the hemisphere limited by the aperture of the stop are virtually the same, the area being small).

Thus, we may express the above relation by

$$\frac{2\,\pi\,F^2}{\pi\,(\frac{1}{2}D)^2} = \frac{8\,F^2}{D^2}$$

This is one of the exposure factors, because the smaller is D, the larger the fraction becomes, and consequently increased exposure. The other factor is $\frac{I}{O}$, for the exposure will vary in this ratio; hence—

$$\text{Exposure factors} = \frac{8\,F^2\,I}{D^2\,O}$$

Taking K and C into account—

$$e = 8\,K\,C\,\frac{F^2\,I}{D^2\,O} \text{ and } E = \frac{2\,K\,C\,F^2\,I}{15D^2\,O}$$

Since $\frac{O}{I} = n^2 \therefore \frac{I}{O} = \frac{1}{n^2}$, and, as well known,

$$F = (n+1)f, \therefore e = 8\,K\,C\left\{\frac{n+1}{n}.\ \frac{f}{D}\right\}^2$$

$$\text{and, since } \frac{f}{D} = \frac{1}{R},$$

$$e = 8\,K\,C\left\{\frac{n+1}{n\,R}\right\}^2$$

Let C be made 1 second, and K = 2, that is loss = 50 per cent., made up thus: say, 10 to 15 per cent. for absorption and reflection of light in passing through the lens, and 30 to 40 per cent. for loss of light due to the ground glass or other material interposed between light and negative.

$$\text{Then } e = 16\left\{\frac{n+1}{n\,R}\right\}^2$$

$$= \left\{4.\ \frac{n+1}{n\,R}\right\}^2$$

To find E, let $\frac{16}{60}$ be taken as $\frac{1}{4}$, which is near enough for practice.

$$\text{Then } E = \left\{\frac{n+1}{2\,n\,R}\right\}^2$$

This formula gives results in minutes, and is easy to apply as well as to remember.

When condensers are employed with a powerful source of light, C becomes a small fraction, ·7 or even ·01. The best way to proceed in such cases is the following:—Obtain the correct exposure in seconds, for one case, by experiment, and let this be called e';

$$\text{Then } e' = 8\,K\,C\left\{\frac{n+1}{n\,R}\right\}^2$$

In this method of working, the loss by absorption may be taken as 20 per cent.

Thus, $K = 1{\cdot}2$, and

$$e' = C\,9{\cdot}6\left\{\frac{n+1}{n\,R}\right\}^2 = \text{approx. } 10\,C\left\{\frac{n+1}{n\,R}\right\}^2$$

$$\text{and } C = \frac{e'}{9{\cdot}6}\left\{\frac{n\,R}{n+1}\right\}^2 = \text{approx. } \frac{e'}{10}\left\{\frac{n\,R}{n+1}\right\}^2$$

Since the values for e', n, and R are known, a known value is found for C. This once found, C becomes a constant for the condenser and lamp with which the experiment was made. The density of the negative has to be taken into account, but, if a medium negative be used for this experiment, very little judgment is required to make the necessary allowance. Or the values for C may be obtained with two or three negatives of varying densities, so as to eliminate "judgment" altogether.

Then with condenser,

$$e = \text{approx. } 10\,C\left\{\frac{n+1}{n\,R}\right\}^2$$

$$\text{and } E = \text{approx. } \frac{C}{6}\left\{\frac{n+1}{n\,R}\right\}^2$$

The value for C just found can be put into these last two equations, but n will probably not have same value in the expression for C as in the one for e; if n has same value in each case, evidently $e = e'$.

If the stops are marked on the decimal system, then since $R = \dfrac{1}{\sqrt{10\,S}}$,

$$\text{Ordinarily, } e = K\,C\,S\left\{9.\frac{n+1}{n}\right\}^2 \text{ approx.}$$

$$\text{For condenser, } e = C\ S\left\{10.\ \frac{n+1}{n}\right\}^2 \text{ approx.}$$

AREA ENLARGING TABLE.

TO ENLARGE FROM

TO ↓	3¼ × 3¼ Lens to Easel.	3¼ × 3¼ Lens to Neg.	4¼ × 3¼ Lens to Easel.	4¼ × 3¼ Lens to Neg.	5 × 4 Lens to Easel.	5 × 4 Lens to Neg.	6½ × 4¾ Lens to Easel.	6½ × 4¾ Lens to Neg.	8½ × 6½ Lens to Easel.	8½ × 6½ Lens to Neg.	10 × 8 Lens to Easel.	10 × 8 Lens to Neg.	12 × 10 Lens to Easel.	12 × 10 Lens to Neg.	15 × 12 Lens to Easel.	15 × 12 Lens to Neg.
3¼ × 3¼	2	2														
4¼ × 3¼	2·1	1·9	2	2												
5 × 4	2·4	1·71	2·1	1·9	2	2										
6½ × 4¾	2·6	1·62	2·4	1·71	2·2	1·83	2	2								
8½ × 6½	3·3	1·43	3	1·5	2·7	1·59	2·3	1·77	2	2						
10 × 8	3·7	1·37	3·4	1·41	3	1·5	2·6	1·62	2·2	1·83	2	2				
12 × 10	4·3	1·3	3·9	1·34	3·5	1·4	3	1·5	2·5	1·66	2·2	1·83	2	2		
15 × 12	5	1·25	4·6	1·28	4	1·33	3·5	1·4	2·8	1·55	2·5	1·66	2·2	1·83	2	2
18 × 16	6·1	1·2	5·5	1·22	4·8	1·26	4·1	1·32	3·3	1·43	2·9	1·53	2·5	1·66	2·2	1·83
20 × 16	6·4	1·18	5·8	1·21	5	1·25	4·3	1·3	3·5	1·4	3	1·5	2·7	1·59	2·3	1·77
22 × 18	7	1·17	6·3	1·2	5·5	1·22	4·6	1·28	3·7	1·37	3·3	1·43	2·8	1·55	2·5	1·66
24 × 18	7·1	1·16	6·6	1·18	5·7	1·21	4·8	1·26	3·8	1·35	3·4	1·41	2·9	1·53	2·6	1·62
24 × 20	7·6	1·15	6·9	1·17	5·9	1·2	5	1·25	4	1·33	3·5	1·4	3	1·5	2·7	1·59
26 × 20	7·9	1·14	7·1	1·16	6·1	1·19	5·2	1·24	4·1	1·32	3·6	1·38	3·1	1·43	2·8	1·55
30 × 24	9·1	1·12	8·2	1·14	7	1·17	5·9	1·2	4·6	1·28	4	1·33	3·5	1·4	3	1·5
36 × 30	10·8	1·1	9·8	1·11	8·4	1·13	7	1·17	5·5	1·22	4·5	1·28	4	1·33	3·5	1·4
42 × 36	12·7	1·08	11·4	1·1	9·7	1·11	8·2	1·14	6·2	1·19	5·4	1·22	4·5	1·28	3·9	1·34

When result is found, multiply each distance by equivalent focal length of lens employed. Thus, if it is desired to enlarge from a ¼-plate negative to 12 x 10 with a lens having 10 inches equivalent focus, then the results from table give 3·9 and 1·34; multiply each by 10, and easel must be placed 39 inches from lens, and negative to lens 13·4 inches.

The easel carries the sensitised paper or the plate.

It frequently happens that a negative enlarged or reduced cannot exactly occupy a given size of plate. In these cases, a simple inspection will determine the best to adopt.

ENLARGEMENT FORMULÆ.

$(n + 1) f$ is the distance of easel from the optical centre of lens, and $\left(\frac{1}{n} + 1\right) f$ the distance of negative from the optical centre of lens. n = number of times enlarged, and f = equivalent focal length of lens employed. For area enlargement,

$$n = \sqrt{\text{number of times area enlarged.}}$$

These formulæ are of conjugate foci, and can therefore be used for problems involving these functions, and for direct enlarging.

TABLE FOR LINEAR ENLARGING.

NUMBER OF TIMES.

1	2	3	4	5	6	7	8
2—2	3—$1\frac{1}{2}$	4—$1\frac{1}{3}$	5—$1\frac{1}{4}$	6—$1\frac{1}{5}$	7—$1\frac{1}{6}$	8—$1\frac{1}{7}$	9—$1\frac{1}{8}$

9	10	11	12
10—$1\frac{1}{9}$	11—$1\frac{1}{10}$	12—$1\frac{1}{11}$	13—$1\frac{1}{12}$

Multiply the above figures by equivalent focal length of lens used. Thus, to enlarge 4 times with a lens of 6 inches focal length, the easel must be placed 30 inches from lens, and the negative $7\frac{1}{2}$ inches.

EQUIVALENT FOCAL LENGTHS
of Dallmeyer's Rapid Rectilinear Lenses (in inches).

$4\frac{1}{4} \times 3\frac{1}{4}$	5×4	6×5	$8\frac{1}{2} \times 6\frac{1}{2}$	10×8	12×10
4	6	$8\frac{1}{4}$	11	13	16

13×11	15×12	18×16	22×20	25×21
$17\frac{1}{2}$	$19\frac{1}{2}$	24	30	33

EQUIVALENT FOCAL LENGTHS
of Ross's Rapid Symmetrical Lenses (in inches).

$4 \times 3\frac{1}{4}$	5×4	6×5	8×5	$8\frac{1}{2} \times 6\frac{1}{2}$	9×7	10×8
$4\frac{1}{2}$	6	$7\frac{1}{2}$	$8\frac{1}{2}$	11	12	13

12×10	13×11	15×12	18×16	22×18	25×22
16	18	20	24	30	34

ENLARGING AND REDUCING

EXPOSURES.

In the previous chapter the formulæ we obtained were :—

For general work, $e = 8\,\text{K}\,\text{C}\left(\frac{n+1}{n\,\text{R}}\right)^2$ (1)

For condenser, $e = 9{\cdot}6\,\text{C}\left(\frac{n+1}{n\,\text{R}}\right)^2$ (2)

These formulæ were intended chiefly for use with artificial light, and probably the majority of photographers enlarge and reduce by this method; consequently, Equation (2) alone will be dealt with, and 9·6 for practical purposes may be written 10, thus we have—

$$e = 10\,\text{C}\left(\frac{n+1}{n\,\text{R}}\right)^2 \qquad (3)$$

The symbols represent the following :—

e = exposure in seconds.

C = contact exposure, *i.e.*, supposing the sensitive plate to be placed against the negative, with no medium between the light and the negative.

n = number of times height of image on sensitive plate is contained in the height of the negative.

R = ordinary "ratio" as marked on stop.

To put Equation (3) in common language, the exposure in seconds is found by the following operation :—"To the number of times height of image is contained in height of negative add 1, multiply this by the denominator of stop number used (in terms of the ratio), and divide the number found by the number of times the height of the image is contained in height of negative; square the result, then multiply this by 10, and finally multiply by the value of the contact exposure."

This appears complicated, but it is really quite simple. Take an example :—Suppose height of image is contained three times in the height of the negative, as in the case of making a lantern plate from a 10 × 8 negative. Let contact exposure, using a condenser, be $\frac{1}{50}$ of a second. Let ratio be

f-16. Then the operations will be thus ;—To 3 add 1, this makes 4 ; multiply by 16 and we get 64 ; divide by 3 and the result is approximately 21 ; square, we then have 441 ; multiply by 10 and we have 4410 ; multiply by $\frac{1}{50}$, and the result is the required exposure in seconds, namely, 88.

A little consideration will show how difficult it is to reduce and enlarge with correct exposures when the lenses are being changed frequently and different stops employed. The writer has generally observed that those persons who perform much of this class of work stick to one lens, and the aperture is never altered. Experience taught them the proper exposure, after a few failures, so all goes well, especially when the pictures to be made follow in certain stereotyped sizes. But it is well known that one particular lens with the same aperture is not suited for all classes of work. If the ordinary worker were to change his lens or the stop, he would probably be "at sea." But the knowledge of this formula would enable him to change about his lenses and stops, and yet give the exposures with the same certainty of obtaining good results as in the "rule-of-thumb" method.

The value of C (the contact exposure) should be determined experimentally, once and for all, for every condenser in the photographer's possession, and with two or three negatives of different densities. The artificial source of illumination will, probably, always be the same in any particular studio, and suppose three different sizes of condensers are in use, then by obtaining results with each condenser from three negatives of different densities, and noting down the nine exposures, nine constants can be obtained for future use. Not one failure ought ever to occur, no matter what lens or stop is used, where the formula given is employed. The question arises how to find these constants, which are the contact exposures. To give one instance :—With any lens and any aperture, knowing the ratio, make trials until a successful enlargement or reduction has been made, noting the exposure. From this result the contact exposure has now to be calculated. Repeat the experiment with one or two more negatives having different densities, and the contact exposures for several densities of negatives can then be obtained. This proceeding eliminates that doubtful element called "judgment."

The value of contact exposure is :—

$$C = \frac{e}{9{\cdot}6}\left(\frac{n\,R}{n+1}\right)^2 \text{ or approximately } \frac{e}{10}\left(\frac{n\,R}{n+1}\right)^2$$

Putting this in common language, "multiply the number of times height of image is contained in height of negative by the ratio ; divide this result by 1 added to the number of times

height of image is contained in height of the negative; square, and divide result by 10, and multiply by the exposure which produced successful result."

Consider afresh the example given.

Proceed thus :—3 multiplied by $\frac{1}{16}$ equals $\frac{3}{16}$; divide by 1 added to 3, or 4, and the result is $\frac{3}{64}$; square, and we get $\frac{9}{4096}$; divide by 10, the result is $\frac{9}{40960}$; multiply by exposure (say 90 seconds), and we obtain result sought $\frac{810}{40960}$, which is approximately $\frac{1}{50}$ of a second.

This is what we assumed in the first example.

Suppose that the stops are marked upon the decimal standard system, which is found in the case of Dallmeyer's lenses, then the formula becomes simplified :—

$$e = \mathrm{C\ S}\left(10\frac{n+1}{n}\right)^2$$

S being the number found on the stop employed.

To give this formula in words :—"To number of times height of image is contained in the height of negative add 1, and divide by number of times height of image is contained in height of negative, multiply result by 10, and square; multiply this result by stop number, and again by contact exposure."

Consider the same example again :—

f-16 by decimal system is 25·6 (*i.e.*, $16^2 \div 10$), we therefore proceed thus :—To 3 add 1, this makes 4; divide by 3 and result is $\frac{4}{3}$; multiply by 10, result is $\frac{40}{3}$; square, and we get $\frac{1600}{9}$, or approximately 180; multiply by 25·6, and we have approximately 4600; multiply this by $\frac{1}{50}$, and the answer is 92.

This answer gives an exposure of four seconds longer than by the other stop notation. The difference is small and of no practical consequence. It arises from the fact that approximate values have been taken in place of true values, in order to deal with simpler figures; and for the photographic purposes to which these formulæ are to be applied this course is quite allowable.

There is a matter which must not be overlooked, namely, when the stops are marked by value of R the exposures will vary according to the square of R, but when marked in accordance with the decimal system the exposure varies simply with the stop number. This is a great advantage. Of course this latter remark holds good when the stops are marked on the Photographic Society's standard, but most makers appear to put the value of R upon their stops, and consequently the formula involving the ratio value is given.

The value of n will be less than unity when enlarging, and greater than unity when reducing.

To those who will take the trouble to master the formulæ just explained, a great reward is in store by saving the expense incurred by numerous failures as well as vexation, and the amateur who takes up the special field of enlarging and reducing as well as making lantern slides will find the method given for correct exposures of the greatest service.

EXAMPLES.

A lantern-plate has to be made from a 10 × 8 negative. Arrange matters so that C = 1 second. Everything being properly adjusted, find exposure in minutes: n may be taken as 3. No condenser used.

$$\text{Let } f = 8 \text{ inches,}$$

$$\text{and } D = \tfrac{3}{4} \text{ inch.}$$

$$\text{Then } E = \left(\frac{1}{2} \cdot \frac{3+1}{3} \cdot \frac{8}{\frac{3}{4}}\right)^2 = \left(\frac{2}{3} \cdot \frac{32}{3}\right)^2 = \frac{64^2}{9^2}$$

$\frac{64}{9}$ may be taken as 7 for a practical result.

Then exposure is 49 minutes.—*Answer.*

If C = 2, 3, &c., then the result must be multiplied by 2, 3, &c.

A lantern-plate has to be made from a 10 × 8 negative. Arrange matters so that C = 1 second. Everything being properly adjusted, find exposure in minutes: n may be taken as 3. No condenser used.

Let $R = \frac{1}{16}$;

$$\text{Then } E = \left(\frac{1}{2} \cdot \frac{3+1}{3} \cdot 16\right)^2 = \left(\frac{32}{3}\right)^2 \text{ or 113 approx.}$$

Then exposure is 113 minutes.—*Answer.*

If C = 2, 3, &c., then the result must be multiplied by 2, 3, &c.

If a condenser had been used, and say C = ·1 second, then we have

$$e = 10 \times \cdot 1 \left(\frac{3+1}{3} \cdot 16\right)^2 = 7 \text{ min. 35 sec., say.}$$

CONDENSERS.

CONDENSERS have two uses. Consider one at a time. When employed in connection with a negative for enlarging or reducing, or with a lantern slide, the function of the condenser is to condense as much light as possible upon the negative or slide. This question is best examined by means of the following diagrams.

Let A represent the source of light.
,, C ,, condenser.
,, L ,, lens employed to form the image.
,, I ,, negative or slide.
,, a ,, angle of rays.

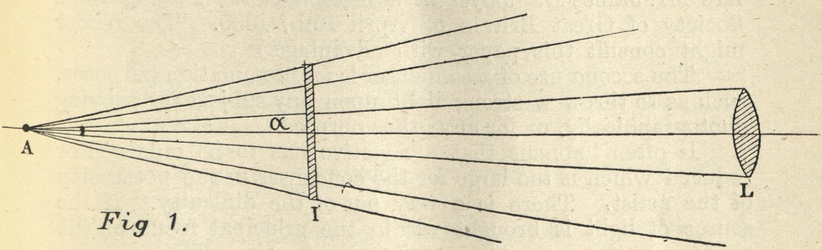


Fig 1.

Figure 1 represents a negative arranged for enlarging or reducing without a condenser. It is clear that only a portion of the rays which fall upon the plate pass the lens, and only a part of the plate will be properly illuminated.

The source of light A must be placed a considerable distance from I, or it will be focussed, more or less distinctly, upon the focussing screen.

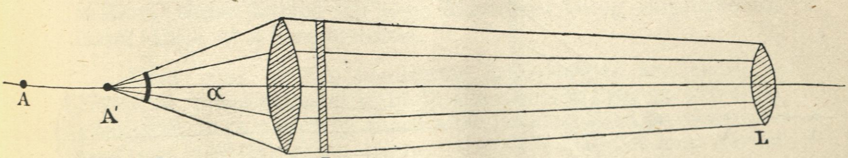


Fig 2.

In Figure 2 a condenser is placed behind the plate. This enables the source of light A being moved nearer the plate to A′; A being its position in Figure 1, so that more light will fall upon the plate for this reason alone. Here also the whole of the rays collected upon the condenser pass through the plate and

through the lens, if the distances have been suitably adjusted. The illumination of the plate will also be equal at all parts.

It will be found that, when the cone of rays does not accurately cover the lens, the correction for achromatism in the lens is destroyed: the result is that the visual and chemical foci are no longer coincident. Lantern plates made under these conditions will appear right to the eye, but when enlarged upon the screen it will not be possible to get the picture sharp. Attention to the proper adjustment of the condenser is therefore of great importance.

Theoretically, the source of light should be a point to produce perfect results.

In practice, a compound condenser is used in the place of a single lens, whereby achromatism is obtained. The lens L is also replaced by any suitable photographic combination. (An excellent communication on this subject was read by the late Mr. James Dallmeyer at a meeting of the Photographic Society of Great Britain on April 16th, 1880. The reader might consult this paper with advantage.)

The second use of a condenser is for illumination purposes, such as to throw a strong light upon any subject for copying photographically, or for any other purpose.

It often happens that a negative has to be enlarged or reduced which is too large for the condenser in the possession of the artist. There is a way out of the difficulty. If the source of light is brought within the principal focus of the condenser, the rays diverge after passing it. The negative must be moved away from the condenser till it is enclosed within the circle of light. The amount of illumination will be much less than if a suitable condenser had been used, but the lighting will be fairly equal over the plate, and the exposure prolonged, but not to an unreasonable extent. The following diagram (Figure 3) shows the method:—

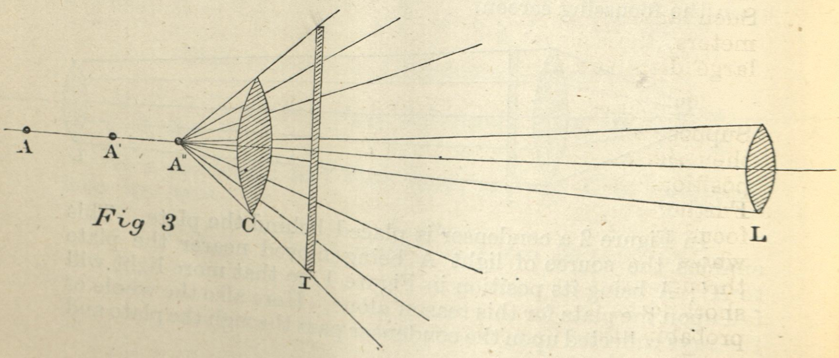


Fig 3

There is sometimes a difficulty to focus the image under this condition of things, but when it occurs a piece of strained tissue paper or a piece of ground glass placed behind the negative settles the trouble. The illumination of the plate will be less as the circle of light where the negative is placed is larger than the condenser area, that is, as the squares of the diameters of these circles.

This case reduces matters to the one given in Figure 1, only the light has been approached to the condenser, and consequently to the lens also. On the other hand the plate I has been moved away from the condenser.

If P = distance of A to L (in Fig. 3).

,, p = distance of A″ to L ,,

,, D = diameter of C ,,

,, d = diameter of I ,,

Since P is invariably greater than p (read case of Fig. 1), we obtain, first, that the exposure in Fig. 3 will be shorter than in Fig. 1 as $\frac{p^2}{P^2}$; and, secondly, the exposure in Fig. 3 will be longer than in Fig. 1 as $\frac{d^2}{D^2}$; therefore total exposure will be shorter (or sometimes longer) in Fig. 3 compared with Fig. 1 as $\frac{d^2\,p^2}{D^2\,P^2}$, and the illumination will be more equal.

In the best forms of optical lanterns, the condenser has an adjustment whereby it can be placed nearer to or farther from the slide: generally two positions are only possible. Such lanterns possess a series of lenses having various diameters, those having the large diameters being employed for large discs at a long distance.

The uses of the condenser adjustments are these:— Suppose the condenser to be in the position nearest the slide, then the rays converge more after passing than in the other position, assuming the source of illumination is not moved. Further suppose that the lens is one of those having a short focus used for throwing a disc at short distances, and that the whole cone of rays proceeding from the condenser just passes through it; then we have the best conditions. If now the short focus lens is changed for one having a longer focus, and probably also larger in diameter, the best conditions are lost,

and the cone of rays does not cover the lens; but if the condenser is shifted to its second position, away from the slide, the result aimed at is secured. Clearly also this last position of the condenser would be unsuitable for the short focus lenses, for part of the light would fall outside these lenses.

Thus, when long focus lenses are used in such a lantern as here referred to, the condenser should be placed in the position farthest from the slide; and for short focus lenses it should be placed nearest the slide.

Apart from the uses of the adjustments just stated, there is another one. It gives the means with lenses of different diameters, when employed for throwing discs at short or long distances, to obtain the whole cone of rays passing the condenser within the area of the lens. The best position for the condenser in these cases is most rapidly found by trial.

A different convergence of the rays can also be obtained by altering the distance between the source of light and the condenser.

RULE FOR ASCERTAINING SIZE
OF DISC FOR LANTERN WORK.

Let O = diameter of disc in inches.
,, $\triangle$ = distance between optical centre of lens and screen in inches.
,, f = equivalent focal length of lens in inches.
,, n = number of times the diameter of slide is contained in the diameter of disc.
,, d = diameter of lantern slide in inches.

Then, since (according to the usual formula) $\triangle = (n + 1)f$ for distance between optical centre of lens and screen,

$$n = \frac{\triangle}{f} - 1$$

Then, if diameter of slide is d, since $\mathrm{O} = n\, d$,

$$\mathrm{O} = \left(\frac{\triangle}{f} - 1\right) d \quad \text{inches};$$

$$\text{and} \quad \mathrm{O} = \left(\frac{\triangle}{f} - 1\right) \frac{d}{12} \quad \text{feet.}$$

$$= \frac{\triangle}{f} \cdot \frac{d}{12} - \frac{d}{12}$$

It is therefore seen that the right-hand side of the equation varies with $\triangle$ so long as the same lens and the same size of slides are used. The calculation is therefore brought down to the utmost simplicity, namely, of

finding the value $\frac{\triangle}{f}$ in any particular case, multiplying this by a constant $\frac{d}{12}$, and from result deducting a constant $\frac{d}{12}$.

Now, to put this in practice :—

Let d = 3 inches (being the most usual size);

$$\text{then} \quad \mathrm{O} = \frac{\triangle}{f} \cdot \frac{1}{4} - \frac{1}{4} \quad \text{feet};$$

$$\tfrac{1}{4} \text{ foot} = 3 \text{ inches};$$

$$\therefore \quad \text{disc} = \frac{\triangle}{f} \cdot \frac{1}{4} \text{ feet, less 3 inches.}$$

In practice, f is known, and the size of lantern slide is about 3 inches diameter. The question to be settled is, for a stated distance between the lantern lens and the screen what will be the size of the disc? The foregoing formula gives the answer. Putting this in words :—" Divide the given distance in inches by focal length of lens in inches, and take a quarter

of this value, then result is the diameter of the disc in feet, if 3 inches are subtracted."

We will proceed to the converse. Suppose the size of the disc is given, also focal length of lens, and the slides are 3 inches diameter, then what distance should the lantern be placed from the screen?

From the foregoing equations the following value is obtained:—

$$\triangle = \frac{1}{12}\left(\frac{O}{3} + 1\right) f \text{ feet.}$$

This form of stating the value is chosen on account of convenience for calculations. Putting the distance in words:—"Divide the size of disc in inches by 3, add 1, multiply the number found by focal length of the lens in inches, and divide result by 12; this is the distance between the lens and screen in feet."

Lastly, given the size of disc required and the distance, what focal length lens should be used? From the equations above,

$$f = \frac{3\ \triangle}{O + 3} \text{ inches.}$$

In words: the focal length of the lens in inches which should be employed to obtain the desired result is found by "multiplying the given distance in inches by 3, and divide result by 3 added to the given diameter of disc in inches."

The most usual lenses in use with the lantern are those having 7, 9, and 12 inch focal lengths; also a metal mask of 3 inches diameter is generally employed to bring all the lantern slides to one size of disc. The table which follows is therefore one which is most useful in practice; the figures have been calculated by the method given above:—

Table of Distances and Sizes of Discs with 6″, 7″, 8″, 9″, 10″, and 12″ Focal Length Lenses.

Distance between Lens and Screen.	FOCAL LENGTH OF LENSES.					
	6″	7″	8″	9″	10″	12″
′	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
15	7·3	6·3	5·4½	4·9	4·3	3·6
20	9·9	8·3	7·3	6·5	5·9	4·9
25	12·3	10·6	9·1½	8·1	7·3	6·0
30	14·9	12·7	11·0	9·9	8·9	7·3
35	17·3	14·9	12·10½	11·5	10·3	8·6
40	19·9	16·11	14·9	13·1	11·9	9·9
45	22·3	19·0	16·7½	14·9	13·3	11·0
50	24·9	21·2	18·6	16·5	14·9	12·3

SIZE OF DISCS.

DEPTH OF FOCUS.

ARTICLE I.

ALTHOUGH "depth of focus" does not really exist, yet, when points in the object are represented in the image by circles having a diameter not exceeding $\frac{1}{100}$th of an inch, the picture appears sharp when viewed in the usual manner. "Depth of focus" may therefore be defined as the distance "between two planes outside the camera, which are so placed that the points in all objects situated in the space between these two planes are represented in the image by circles not exceeding $\frac{1}{100}$th of an inch in diameter."

We will now proceed to find a set of formulæ by which this depth of focus may be measured for all cases. A flat field need not be presumed, but for the required definition to appear upon the plate this is necessary, otherwise the plate must be bent to the shape of the field.

The common equations for expressing the relation between conjugate foci are $O = (n + 1) f$ and $I = \left(\frac{1}{n} + 1\right) f$, where f = solar focus, $n f + f$ = distance of the object from the optical centre of the lens, O = object, and I = image.

It is supposed that the object is not enlarged, but it is evident that the following propositions are good in this case also, it being simply necessary to change the places of object and image.

Let $\triangle$ = distance of object from optical centre of lens, and therefore represents a conjugate focus.

Let $\triangle_1$ and $\triangle_2$ = respectively the farthest and nearest distance from the optical centre of the lens, where, if a plane object is placed so that the optic axis is at right angles to it, points in it are represented in the image by circles no greater than $\frac{1}{100}$th of an inch.

Let D = depth of focus, *i.e.*, $\triangle_1 - \triangle_2$.

,, S = decimal standard marked on stop.

,, R = ratio as marked upon the stop, for expressing relation between the aperture and equivalent focus, *i.e.*, for parallel rays.

,, R_1 = the actual ratio, as expressed by the aperture divided by the conjugate within the camera.

,, d = conjugate within the camera.

,, a = aperture.

Then we have the following equations:—

$$D = \triangle_1 - \triangle_2 \qquad (1)$$

$$\triangle = (n + 1) f \qquad (2)$$

$$d = \left(\tfrac{1}{n} + 1\right) f \qquad (3)$$

Also, because

$$\triangle = (n + 1) f$$

$$n = \frac{\triangle - f}{f} \qquad (4)$$

and

$$\frac{1}{n} = \frac{f}{\triangle - f} \qquad (5)$$

Again,

$$R = \frac{a}{f} \text{ for solar focus,}$$

and for all other conjugates the camera extension is greater than f; this must therefore be replaced by d, and the ratio is R_1, hence $R_1 = \frac{a}{d}$, and from Equations 3 and 5 we have

$$d = \left(\tfrac{1}{n} + 1\right) f = \left(\frac{f}{\triangle - f} + 1\right) f$$

$$\therefore \quad R_1 = \frac{a}{d} = \frac{a}{\left(\frac{f}{\triangle - f} + 1\right) f} = \frac{a\,(\triangle - f)}{f\,\triangle}$$

$$\text{and since } \frac{a}{f} = R$$

$$\text{we have } R_1 = \frac{R\,(\triangle - f)}{\triangle} \qquad (6)$$

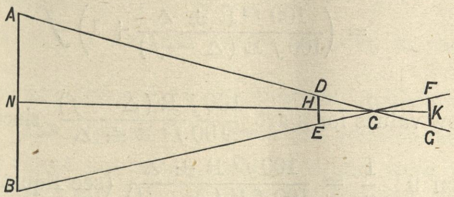
We now require to know how much the focussing glass may be moved towards or away from the lens in order to comply with the definition of depth of focus. We must first consider the conditions which control these distances.

Evidently the movement may be greater as the rays tend to become more parallel within the camera—that is, the greater the distance from the lens at which they are brought to a focus the nearer parallel they become. Parallelism also increases with reduction of aperture. Hence, parallelism of the rays inside the camera increases with longer focus and decreases with an increased aperture. This may be expressed by saying the distance through which the screen may be moved varies

as $\frac{f}{a}$, and since $\frac{a}{f}$ here is the same as R_1, for the value of f is d, we have

$$\therefore \frac{f}{a} = \frac{d}{a} = \frac{1}{R_1} = \frac{\triangle}{R(\triangle - f)} \text{ (by Equation 6)} \quad (7)$$

The limit of the movement can be ascertained from the following figure :—



Let AB = aperture = a; NC = conjugate within camera = d; DE and FG = the places to which the screen may be moved from C, so that DE and FG each equal $\frac{1}{100}$th inch.

HC and KC = these distances (which are equal) = δ

Then from similar triangles,

$$\frac{NC}{AB} = \frac{HC}{DE} = \frac{KC}{FG} \quad i.e. \quad \frac{d}{a} = \frac{\delta}{\frac{1}{100}} = 100\,\delta$$

$\therefore \ \delta = \frac{d}{100\,a}$ Putting in the value for $\frac{d}{a}$ found by Equation 7, and considering the distance from C towards A B positive, and from C towards F G negative, we have—

$$\delta = \pm \frac{\triangle}{100\,R(\triangle - f)} \qquad (8)$$

which is the distance to which the screen may be moved on either side of C, the plane of true focus.

When $d = f$, $\delta = \frac{1}{100\,R}$, $\because \frac{d}{a} = \frac{f}{a} = \frac{1}{R}$. Under these conditions the negative value does not exist, because no image can be formed within the principal focus.

Now the conjugate within the camera is by hypothesis $\left(\frac{1}{n} + 1\right) f$; substituting the value found in Equation 5 for $\frac{1}{n}$ we obtain

$$d = \left(\frac{f}{\triangle - f} + 1\right) f$$

Now d is to be increased or diminished by $\frac{\triangle}{100\,R(\triangle - f)}$ (see Equation 8), $\therefore$ the new values for d, say d_1 and d_2, are—

$$d_1 \text{ and } d_2 = \left(\frac{f}{\triangle - f} + 1\right) f \pm \frac{\triangle}{100\,\mathrm{R}\,(\triangle - f)}$$

(d_1 with + sign, and d_2 with — sign.)

$$= \left(\frac{f}{\triangle - f} \pm \frac{\triangle}{100\,f\,\mathrm{R}\,(\triangle - f)} + 1\right) f$$

$$= \left(\frac{100\,f^2\,\mathrm{R} \pm \triangle}{100\,f\,\mathrm{R}\,(\triangle - f)} + 1\right) f \qquad (9)$$

$\therefore$ new values for n are $\dfrac{100\,f\,\mathrm{R}\,(\triangle - f)}{100\,f^2\,\mathrm{R} \pm \triangle}$ since by form of Equation 9 $\dfrac{1}{n} = \dfrac{100\,f^2\,\mathrm{R} \pm \triangle}{100\,f\,\mathrm{R}\,(\triangle - f)}$ (see Equation 3).

From this we can find the values of $\triangle_1$ and $\triangle_2$, for $\triangle_1$ and $\triangle_2 = (n + 1)\,f$, n having different values for $\triangle_1$ and $\triangle_2$, and when it has its new values—

$$\triangle_1 \text{ and } \triangle_2 = \left(\frac{100\,f\,\mathrm{R}\,(\triangle - f)}{100\,f^2\,\mathrm{R} \pm \triangle} + 1\right) f \qquad (10)$$

($\triangle_1$ with — sign, and $\triangle_2$ with + sign.)

and $\triangle_1$ is greater than $\triangle_2$,

$$\therefore \mathrm{D} = \triangle_1 - \triangle_2 = \left(\frac{100f\mathrm{R}(\triangle - f)}{100\,f^2\,\mathrm{R} - \triangle} + 1\right) f - \left(\frac{100f\mathrm{R}(\triangle - f)}{100\,f^2\,\mathrm{R} + \triangle} + 1\right) f$$

$$= \frac{100\,f^2\,\mathrm{R}\,(\triangle - f)}{100\,f^2\,\mathrm{R} - \triangle} - \frac{100\,f^2\,\mathrm{R}\,(\triangle - f)}{100\,f^2\,\mathrm{R} + \triangle}$$

$$= \frac{100\,f^2\,\mathrm{R}\,(\triangle - f)\,(100\,f^2\,\mathrm{R} + \triangle - 100\,f^2\,\mathrm{R} + \triangle)}{(100\,f^2\,\mathrm{R})^2 - \triangle^2}$$

$$\therefore \mathrm{D} = \frac{2\,(\triangle - f)}{\dfrac{100\,f^2\,\mathrm{R}}{\triangle} - \dfrac{\triangle}{100\,f^2\,\mathrm{R}}} \qquad (11)$$

If for $\triangle$ we put its value $(n + 1)\,f$, we obtain—

$$\frac{2\,(\triangle - f)}{\dfrac{100\,f^2\,\mathrm{R}}{\triangle} - \dfrac{\triangle}{100\,f^2\,\mathrm{R}}} = \frac{2\,(n\,f + f - f)}{\dfrac{100\,f^2\,\mathrm{R}}{(n+1)\,f} - \dfrac{(n+1)\,f}{100\,f^2\,\mathrm{R}}} = \frac{2\,n\,f}{\dfrac{100\,f\,\mathrm{R}}{n+1} - \dfrac{n+1}{100\,f\,\mathrm{R}}} \qquad (12)$$

and since $R = \frac{1}{\sqrt{10 \quad S}}$ where S = stop numbers expressed by the decimal system, and $\sqrt{10}$ = approximately 3·16, we have (from Equation 12)

$$D = \frac{2\,n\,f}{\frac{f}{\cdot 0316\,(n+1)\,\sqrt{S}} - \frac{\cdot 0316\,(n+1)\,\sqrt{S}}{f}} \qquad (13)$$

and values for f, n, &c., can easily be found in the terms of the rest.

Let us now proceed to the practical applications of the foregoing. The expression $\frac{1}{n}$ gives the relation between the size of the object and image; and in studio work n is generally between 1 and 10. Hence, the fraction occurring in Equation 13, $\frac{\cdot 0316\,(n+1)\,\sqrt{S}}{f}$ rarely exceeds $\frac{\cdot 3476\,\sqrt{S}}{f}$ or approximately $\frac{\sqrt{S}}{3\,f}$; again, S in the studio is seldom larger than 25, and f is usually greater than 8, so that $\frac{\sqrt{S}}{3\,f}$ is seldom larger than $\frac{1}{5}$. As a rule, the fraction has a much smaller value, so that for portraiture we may neglect the fraction and write Equation 13 so:

$$D = \frac{2\,n\,f}{\frac{10000\,f}{316\,\sqrt{S}\,(n+1)}} = \frac{2\,n\;316\,\sqrt{S}\,(n+1)}{10000}$$

by reduction $\quad D = \cdot 0632\,n\,(n+1)\,\sqrt{S} \qquad (14)$

and expressed in terms of R, since $S = \frac{1}{10\,R^2}$

$$D = \frac{n\,(n+1)}{50\,R} \qquad (15)$$

Again, in the same manner we may obtain—

$$S = \frac{250\,D^2}{[n\,(n+1)\,]^2} \qquad (16)$$

and $$R = \frac{n\,(n+1)}{50\,D} \qquad (17)$$

also $$n = \sqrt{\frac{D}{\cdot 0632\,\sqrt{S}} + \tfrac{1}{4}} - \tfrac{1}{2} \qquad (18)$$

and $$n = \frac{\sqrt{200\,R\,D + 1} - 1}{2} = \text{approx. } 7\,\sqrt{R\,D} - \tfrac{1}{2} \qquad (19)$$

and
$$f = \frac{A \sqrt{D}}{\sqrt{100\,D - 20\,A\,n}} \qquad (20)$$
$$\text{when } A = \cdot 316\,(n+1)\,\sqrt{S} \text{ or } \frac{n+1}{10\,R}$$

and 100 D must be greater than 20 A n to obtain a rational value. The value for f is rarely required, for the lenses ready constructed are in the possession of the operator.

When copying equal size, $n = 1$, and Equations 14 and 15 become

$$D = \cdot 126\,\sqrt{S} = \frac{\sqrt{S}}{8} \text{ approx.} \qquad (21)$$
$$D = \frac{1}{25R} \qquad (22)$$

When in Equation 11,

$$\frac{100\,f^2\,R}{\triangle} = \frac{\triangle}{100\,f^2\,R} \quad \text{the value for } D = \infty;$$

this occurs when $\triangle = 100\,f^2\,R$, and signifies that after the distance $100\,f^2\,R$ all will be in focus, according to our definition. However, this distance is not quite accurate, for it should be $f + 100\,f^2\,R$, as will be shown; but it is well to point out how the value $100\,f^2\,R$ comes about.

We started to find Equation 11 by employing R_1 in terms of R, $\triangle$, and f, because R_1 is not equal to R; but for infinite depth of focus it is evident that this correction is unnecessary, since the image will be formed at $f + \frac{1}{100\,R}$ from the optical centre of the lens.

To proceed to find the value for indefinite depth of focus:

$$d = \left(\frac{1}{n} + 1\right) f; \text{ now add } \frac{1}{100\,R} \text{ (see page 43);}$$

the negative value of $\frac{1}{100\,R}$ cannot be taken, for the image would be formed at a less distance than the solar focus, which is not possible. If the same course is now followed as was done to find Equation 11, the numerator becomes—

$$\frac{100\,f^2\,R}{\triangle - f} - \frac{\triangle - f}{100\,f^2\,R}$$

giving $\triangle = f + 100\,f^2\,R$ for infinite depth of focus.

But there is a simpler method:

We have
$$d = f + \frac{1}{100\,R} = \left(\frac{1}{100\,f\,R} + 1\right) f$$
$$\therefore \quad \triangle = (100\,f\,R + 1)\,f = f + 100\,f^2\,R \qquad (23)$$

Finally, when $\triangle = $ or $< f$, Equation 11 becomes $D = O$, meaning that no depth of focus exists, because the rays after passing the lens become parallel or divergent, and therefore never come to a focus.

To recapitulate, we have obtained the following equations and information :—

A. Depth of focus varies as $\frac{1}{R}$

B. Distance from lens to nearest plane of depth of focus,

$$= f + \frac{100 f^2 R (\triangle - f)}{100 f^2 R + \triangle}$$

C. Distance from lens to farthest plane of depth of focus,

$$= f + \frac{100 f^2 R (\triangle - f)}{100 f^2 R - \triangle}$$

D.

$$D = \frac{2 (\triangle - f)}{\frac{100 f^2 R}{\triangle} - \frac{\triangle}{100 f^2 R}}$$

E.

$$D = \frac{2 (\triangle - f)}{\frac{100 f^2}{3{\cdot}16 \triangle \sqrt{S}} - \frac{3{\cdot}16 \triangle \sqrt{S}}{100 f^2}}$$

F.

$$D = \frac{2 n f}{\frac{100 f R}{n + 1} - \frac{n + 1}{100 f R}}$$

G.

$$D = \frac{2 n f}{\frac{f}{{\cdot}0316 (n + 1) \sqrt{S}} - \frac{{\cdot}0316 (n + 1) \sqrt{S}}{f}}$$

Where $\frac{\triangle}{100 f^2 R} = \frac{3{\cdot}16 \triangle \sqrt{S}}{100 f^2} = \frac{{\cdot}0316 (n + 1) \sqrt{S}}{f}$

is very small, we have as approximate values :

H.

$$D = {\cdot}0632\, n (n + 1) \sqrt{S}$$

I.

$$D = \frac{n (n + 1)}{50 R}$$

J.

$$S = \frac{250 D^2}{[n (n + 1)]^2}$$

K.

$$R = \frac{n (n + 1)}{50 D}$$

L.

$$n = \sqrt{\frac{D}{{\cdot}0632 \sqrt{S}} + \tfrac{1}{4}} - \tfrac{1}{2}$$

M.

$$n = 7 \sqrt{R D} - \tfrac{1}{2}$$

N.
$$f = \frac{A\sqrt{D}}{\sqrt{100\,D - 20\,A\,n}}$$

when $A = \cdot316\,(n+1)\,\sqrt{S}$ or $\frac{n+1}{10\,R}$

and $100\,D > 20\,A\,n$

For equal copying, generally

O.
$$D = \frac{1}{25\,R - \frac{1}{100\,f^2\,R}}$$

P.
$$D = \frac{1}{\frac{1}{\cdot126\,\sqrt{S}} - \frac{\cdot0316\,\sqrt{S}}{f^2}}$$

Q.
$$D = \cdot126\,\sqrt{S}$$

R.
$$D = \frac{1}{25\,R}$$

For infinite depth of focus,

S.
$$\triangle = f + 100\,f^2\,R$$

when $R =$ approx. $\frac{1}{8}$

T.
$$\triangle = \text{approx. } f^2 \text{ feet.}$$

The above twenty expressions and equations give every possible formula connected with depth of focus in accordance with the definition, not only in complete forms but also in simple expressions, quite accurate enough for solving problems of every kind, whether by the Decimal or Photographic Society of Great Britain's standard. Two points, however, may be noticed: firstly, that in general, when the formulæ in which a part of the denominator is neglected becomes inaccurate to an appreciable degree, the complete formula for depth of focus can be used; secondly, the only equation which is troublesome to solve is the one for finding the value of f, and in practice it is only required by the lens maker.

It may also be remarked that if a negative curvature of field can be produced, without introducing other bad effects, except to a very limited extent, for taking all objects on one particular scale, far greater depth of focus may be obtained, and the depth decreases as the positive curvature increases. These statements may appear contradictory, since depth of focus is clearly independent of the nature of the lens; but they are true when applied to the image on a flat glass plate, and are easily appreciated by drawing a diagram.

If $\frac{1}{100}$th of an inch is not to be the standard, the above formulæ must be re-adjusted to the standard proposed.

DEPTH OF FOCUS.

ARTICLE II.

LET us define depth of focus thus:—"The distance between two planes in space wherein all objects appear well defined on the focussing screen." Generally when a point in any object is represented by a circle of confusion having a diameter not exceeding $\frac{1}{100}$th of an inch, it is held that the definition of the image is sufficiently sharp. A flat field is presumed in the case of photographic pictures. The following formulæ are based on these suppositions, but any other hypothesis could be used, should it be desired, and the equations altered to suit. To avoid repetition and details, it will be best to refer to the last article on "Depth of Focus."

Let D = depth of focus.

,, f = equivalent focal length.

,, $\triangle$ = distance of object from optical centre of lens.

,, R = ratio or intensity.

Then it was shown that—

$$D = \frac{2(\triangle - f)}{\frac{100 f^2 R}{\triangle} - \frac{\triangle}{100 f^2 R}} \qquad \text{(I.)}$$

The true focussing plane, outside the camera, is not situated midway, but this does not affect the truth of the formula, or the results derived therefrom. If $\triangle = nf$ where n is a number, then—

$$D = \frac{2f(n-1)}{\frac{100 f R}{n} - \frac{n}{100 f R}} \qquad \text{(II.)}$$

which is simpler to use in practice.

Again, if S = decimal standard marked on the stop, then since—

$$R = \frac{1}{\sqrt{10\ S}}$$

$$D = \frac{2f(n-1)}{\frac{f}{n}\sqrt{\frac{1000}{S}} - \frac{n}{f}\sqrt{\frac{S}{1000}}} \qquad \text{(III.)}$$

Or—

$$D = \frac{2f(n-1)}{\frac{10f}{n}\sqrt{\frac{10}{S}} - \frac{n}{10f}\sqrt{\frac{S}{10}}} \qquad \text{(IV.)}$$

Again, since $\frac{1}{\sqrt{10\,S}} = \frac{1}{\sqrt{10}\,\sqrt{S}}$ and $\sqrt{10} = 3{\cdot}16$ approximately, we have $\frac{1}{\sqrt{10}\,\sqrt{S}} = \frac{1}{3{\cdot}16\,\sqrt{S}}$

By substituting this value in Equation IV., the following values for D are found :—

$$D = \frac{2f(n-1)}{\frac{100f}{3{\cdot}16\,n\sqrt{S}} - \frac{3{\cdot}16\,n\sqrt{S}}{100f}} \qquad \text{(V.)}$$

$$D = \frac{2f(n-1)}{\frac{10f}{{\cdot}316\,n\sqrt{S}} - \frac{{\cdot}316\,n\sqrt{S}}{10f}} \qquad \text{(VI.)}$$

$$D = \frac{2f(n-1)}{\frac{f}{{\cdot}0316\,n\sqrt{S}} - \frac{{\cdot}0316\,n\sqrt{S}}{f}} \qquad \text{(VII.)}$$

$$D = \frac{2f(n-1)}{\frac{31{\cdot}6f}{n\sqrt{S}} - \frac{n\sqrt{S}}{31{\cdot}6f}} \qquad \text{(VIII.)}$$

$$D = \frac{2f(n-1)}{\frac{10fC}{n} - \frac{n}{10fC}} \qquad \text{(IX.)}$$

where $C = \sqrt{\frac{10}{S}}$

All the above equations will be found of service in practice. To assist in the solution of problems, the following square roots to two places of decimals are given because the $\sqrt{S}$ is often required to solve the above equations. Only the general run of stop numbers are given :—

S		$\sqrt{S}$	S		$\sqrt{S}$	S		$\sqrt{S}$
8	—	2·82	14	—	3·74	20	—	4·47
9	—	3·	15	—	3·87	22	—	4·69
10	—	3·16	16	—	4·	25	—	5·
12	—	3·46	18	—	4·24	30	—	5·47

Barlow's tables of squares and square roots are very useful to avoid long calculations.

Before proceeding to the question of problems, let us analyse these equations. Take Equation II.—

$$D = \frac{2f(n-1)}{\frac{100fR}{n} - \frac{n}{100fR}}$$

It is clear that when $n = 1$, $D = O$. In other words, depth of focus cannot exist, because the rays are parallel after passing through the lens.

When copying equal size $nf = 2f \therefore n = 2$, and the equation becomes—

$$D = \frac{2f}{\frac{100fR}{2} - \frac{2}{100fR}} = \frac{2f}{50fR - \frac{1}{50fR}}$$

Or—

$$D = \frac{1}{25R - \frac{1}{100f^2R}}$$

Now $\frac{1}{100f^2R}$ is usually very small, so we obtain the simple form of equation:—

$$D = \frac{1}{25R} = \frac{1}{25\frac{1}{\sqrt{10S}}} = \frac{\sqrt{10S}}{25}$$

$$\therefore \quad D = \frac{3 \cdot 16}{25}\sqrt{S} = \cdot 126\sqrt{S} = \text{approx. } \frac{\sqrt{S}}{8} \qquad \text{(X.)}$$

Again, when $\frac{100fR}{n} = \frac{n}{100fR}$, D = infinity; that is, after a certain distance all will be in focus.

Let us proceed to examine Equation VI.—

$$D = \frac{2f(n-1)}{\frac{10f}{\cdot 316\,n\sqrt{S}} - \frac{\cdot 316\,n\sqrt{S}}{10f}}$$

When f is large, n small, and the stop marked by S to any number generally employed—say up to 30—then $\frac{\cdot 316\,n\sqrt{S}}{10f}$ becomes a very small fraction, and may be neglected for practical purposes; we then obtain—

$$D = \cdot 0632\,n(n-1)\sqrt{S} \qquad \text{(XI.)}$$

$$S = \frac{D^2}{[\cdot 0632\,n(n-1)]^2} \qquad \text{(XII.)}$$

$$n = \sqrt{\frac{D}{0{\cdot}632\ \sqrt{S}} + \frac{1}{4}} + \frac{1}{2} \qquad \text{(XIII.)}$$

f is here eliminated, so its value cannot be found by this method of simplification: but Equation VI. must be reduced in the usual way, giving—

$$f = \frac{A\ \sqrt{D}}{\sqrt{100\ D - 20\ A\ (n-1)}} \qquad \text{(XIV.)}$$

$$\text{where} \qquad A = {\cdot}316\ n\ \sqrt{S}$$

and, since $\sqrt{S}$ and $\sqrt{D}$ are known, the value for f is easily obtained. This equation has one drawback, namely, that, unless most exact values are inserted, the result found for f is very far from the truth, and therefore not convenient in practice.

However, there is no reason to use this formula at all, because photographers have a certain number of lenses by them, and they know beforehand the most suitable ones to employ in any given case. It is therefore only necessary to obtain, by trial with any of the above formulæ, which lens in their possession is best to use. In fact, it is only the value of S, D, and n, which are generally required to be found, and Equations XI., XII., and XIII., are very simple expressions wherewith to find them. The reason why f disappears in these equations is due to the fact that their geometrical representations will be found to consist of sets of nearly similar triangles. Yet it must be remembered that f is not really eliminated if absolutely correct results are desired.

Before proceeding to the application of depth of focus formulæ, we will put some of the most useful equations into a more workable form.

The value of n, as given above, is not the same as the value of n in the usual formulæ $(n + 1) f$, and $\left(\frac{1}{n} + 1\right) f$, so we will arrange that n shall have the same value in all formulæ.

Now in the above equations $\triangle = n\,f$, call this $n'\,f$, and in the common formulæ $\triangle = (n + 1)\,f$,

$$\therefore \quad n'\,f = (n + 1)\,f;$$

hence—

$$n' = n + 1,$$

consequently we must put $n + 1$ for n throughout the given equations, in order that n may always have the same value.

Let us re-write Equations II., VI., XI., XII., XIII., and XIV., substituting $n + 1$ for n.

We shall obtain the following:—

II. becomes—

$$D = \frac{2nf}{\dfrac{10f}{\cdot 316(n+1)\sqrt{S}} - \dfrac{\cdot 316(n+1)\sqrt{S}}{10f}} = \frac{n}{\dfrac{5}{A} - \dfrac{A}{20f^2}} \qquad (a.)$$

VI. becomes—

$$D = \frac{2\ n\ f}{\dfrac{100fR}{n+1} - \dfrac{n+1}{100fR}} \qquad (b.)$$

XI. becomes—

$$D = \cdot 0632\, n\,(n+1)\,\sqrt{S} = \cdot 2\, A\, n \qquad (c.)$$

XII. becomes—

$$S = \frac{D^2}{[\cdot 0632\, n\,(n+1)]^2} = \frac{250\ D^2}{[n\,(n+1)]^2} \qquad (d.)$$

$$\text{and } R = \frac{n\,(n+1)}{50\ D} \qquad (e.)$$

XIII. becomes—

$$n = \sqrt{\frac{D}{\cdot 0632\sqrt{S}} + \frac{1}{4}} - \frac{1}{2} \qquad (f.)$$

$$\text{also } n = 50\sqrt{R\,D + \tfrac{1}{4}} - \tfrac{1}{2} = 7\sqrt{R\,D} - \tfrac{1}{2} \qquad (g.)$$

XIV. becomes—

$$f = \frac{A\sqrt{D}}{\sqrt{100\ D - 20\ A\ n}} \qquad (h.)$$

and in this equation, that a possible value may exist, 100 D must be greater than 20 A n. We have by X. for copying equal—

$$D = \cdot 126\sqrt{S} = \frac{1}{25\ R} \qquad (i.)$$

When using Equations a, c, and h, it is always necessary to see whether

$$\frac{\cdot 316\ (n+1)\sqrt{S}}{10f} = \frac{A}{10f} = \frac{\cdot 001\ (n+1)}{R\ f}$$

is so small that it may be neglected. The same remark applies to the fraction $\dfrac{1}{100\, f^2\, R}$ when employing Equation i (see solution of X.).

We may now proceed to problems.

Problem A.

With a 30″ equivalent focus lens, it is desired to take a 3″ head (9″ being taken as life size), and 3″ depth of focus is required. What stop should be used if marked on the decimal system?

Here $n = 3$, since $\frac{1}{n} =$ the reduction, which is $\frac{1}{3}$ in this case, $f = 30''$ and $D = 3''$.

The fraction $\frac{\cdot 316\,(n+1)\sqrt{S}}{10\,f} = \cdot 00421\sqrt{S}$, evidently this is very small, so we may employ Equation d.

$$\therefore S = \frac{9}{(\cdot 0632 \times 3 \times 4)^2} = \frac{9}{\cdot 7584^2} = \text{approx.}\ \frac{90}{6} = 15\ \textit{Answer}.$$

PROBLEM B.

The same data as in the last problem, but ascertain what depth of focus will be obtained with a stop marked as S. Employ Equation c.

Hence—

$$D = \cdot 0632 \times 3 \times 4\sqrt{20},\ \text{and}\ \sqrt{20} = 4{\cdot}47,$$

$$\therefore\ D = \cdot 7584 \times 4{\cdot}47 = 3{\cdot}39''\ \textit{Answer}.$$

PROBLEM C.

The same lens as in Problem A. What depth of focus if 20 S is used and a life-size head is taken? Here $\frac{1}{100\,f^2\,R}$ is very small, so we may employ Equation i.

Hence—

$$D = \cdot 126\sqrt{20} = \cdot 126 \times 4{\cdot}47 = \cdot 5632\ \therefore D = \text{approx.}\ \cdot 6''\ \textit{Answer}.$$

PROBLEM D.

Equivalent focal length of lens employed is 24″, stop used is marked 18 S. What size head can be taken if focus is to be 4″?

Here $\frac{\cdot 316\,(n+1)\sqrt{S}}{10\,f} = \cdot 005565\,(n+1)$ and, since n will not be large, this fraction has a very small value. We may therefore employ Equation f.

$$\text{Hence } n = \sqrt{\frac{4}{\cdot 0632 \times 4{\cdot}24} + \frac{1}{4}} - \frac{1}{2} = 5{\cdot}1 - \cdot 5\ \text{approx.}$$

$$\therefore\ n = \text{approx.}\ 4{\cdot}6.$$

Since the reduction is expressed by $\frac{1}{n}$, in this case it is $\frac{1}{4{\cdot}6}$ or $\frac{10}{46}$ and, the head being 9″, the image will be $\left(\frac{10}{46} \times 9\right) = \frac{90}{46} =$ say 2″ *Answer*.